New Module by Prof. Dr. Peter Knabner

Mathematical Theory for Simulation Practitioners

This novel course addresses master and PhD students with basic knowledge and experience in continuum mechanics based mathematical modeling and numerical simulations of natural or technical processes. Its goal is to provide or refresh or widen the necessary background information from mathematics, both concerning model formulation and validity and discretization and solution methods. Due to this wide span, the lecture can only touch selected topics, also relying on the active reading activity of the participants. In particular, specific interests and questions of the participants can be addressed in the discussion hours. Examples of questions which may come up and be discussed are

- Is my model thermodynamically consistent ?
- What means mathematically rigorous or formal, what are the consequences ?
- What am I allowed to do with which functions ?
- What is a good discretization method ?
- Is my discretization locally mass conservative, if not, what to do ?
- Etc.

The course will be webex- based, the exact dates will be announced soon. For better planning, you may send my an email (knabner@math.fau.de), indicate your interest and also if necessary indicate which dates are impossible for you.

Course/lectures

Lectures : 2 semester hours/week

Reading discussion: 1 semester hour/week

Target group

Master and PhD students of

Simulation Technology, Applied Mathematics, Engineering, Physics, Computer Science

with decent mathematical knowledge

Content

- *Mathematics of continuum mechanics*

(Conservation laws, constitutive laws, second law of thermodynamics, frame indifference,

viscous fluids, elastic solids)

- *Mathematics of partial differential equation (pde) models*

(Classical, weak, conservative, variational formulations, survey of existence, uniqueness

qualitative properties for elliptic and parabolic equations)

- *Scales of modeling, models with several scales, scale transition*

(Periodic homogenization: formal two-scale expansion vs. rigorous two-scale convergence,

Theory of averaging, mixture theory, particle scales)

- *Approximation methods for linear pde models: Selection criteria*

(Finite difference method, conforming and nonconforming finite element methods, mixed methods: stability via inverse monotonicity, coercivity, or LBB, approximation space error, consistency error, convergence error, orders of convergence, superconvergence, qualitative properties: maximum principles, flux approximation, local mass conservation, postprocessing)

Learning objectives and skills

Students

- derive models for fluid mechanics and elasticity theory,
- evaluate the predictive power of models using physical modelling assumptions and the qualitative characteristics of solutions,
- apply analytical techniques to rigorously prove qualitative properties of solutions,
- apply algorithmic approaches for models with partial differential equations and explain and evaluate them,
- are capable to judge on a numerical method's properties regarding stability and efficiency,
- apply a broad spectrum (focus on conforming finite element methods for parabolic problems), extending these approaches also to nonlinear problems,

Used textbooks

C. Eck, H. Garcke, P. Knabner: Mathematical Modeling, Springer, 2017

P. Knabner, L. Angermann, Numerical Methods for Elliptic and Parabolic Partial Differential Equations, Springer, New York, 1st edition 2003, extended 2nd edition 2021 (will be provided for internal use)