TEAM 1004

Report 1004

How much fuel is required for a manned mission to mars?

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New Zealand’s Next Top Engineering Scientist Competition 2010

Summary

From this investigation, we found out that the mass of fuel required for a manned mission to Mars is **158602.5776 kg**. We analysed the problem and found three sub-problems – 1. How long it would take to get to Mars and back, and the mechanism of getting there, 2. The requirements of the astronaut, and from these two, we got the value for the mass of the fuel.

Since getting to Mars involves escaping the gravitational field of Earth, and getting back also requires this (for Mars), we have taken this into account also. Fuel is needed for this. Other than this, we also found the most fuel efficient pathway for our spacecraft to travel on based on previous findings and Kepler's Laws, and found the amount of fuel required to boost the rocket into this pathway. The mass of the spacecraft decreases during the mission and this is due to taking the behaviour of the astronaut into consideration. In order to make the calculations realistic, we analysed how our rocket should be made, and formed the components.

During solving these problems, we produced a diagrammatic breakdown of the mission into 4 stages, and considered each stage separately, summing up in the end. We expect that the value that we obtained is less than the real value, as many assumptions were made in order to make modelling possible.

Introduction

How much fuel is required for a manned mission to mars?

People have always been fascinated with exploring the unknown. Space – that which is beyond the boundaries of our planet – has become one of the main focuses of our efforts in this technology driven age. It's no surprise then that we have set our eyes towards Mars, a planet that is close to our own and the subject of a lot of our science fiction; its appeal to our inner explorer is powerful. To actually reach Mars is quite a Herculean task and for the purposes of this report, we have chosen to ignore the financial cost in such an endeavour (under the assumption that the funding put into space travel by organisations like NASA will not doubt get us to the red planet anyway). Our solution involves finding the fuel required for a single astronaut to escape from Earth's gravitational field, travel to Mars, land (and explore), escape Mars' gravitational field and finally return to Earth.

Our considerations in preparing an answer to the above question will deal with the following:

1. How long will the trip take?
2. What will the astronaut need, and more importantly, what is the mass of these resources?

And finally,

3. How much fuel will we need to achieve the mission?

Main

In order to answer the question we have been posed, we first need to know the following:

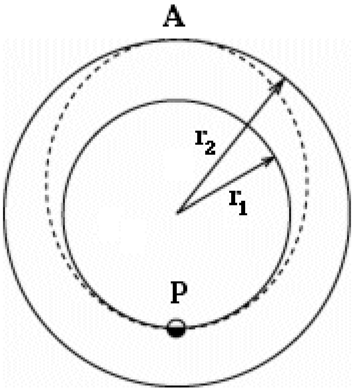
- The length of the trip will affects the amount of resources
- The amount of resources affects the mass
- The mass affects the amount of fuel

1. How long will the trip take?

In ascertaining the ideal duration of the trip we had to factor in the following:

- Relative orbit speeds of Mars and Earth
- The inclusion of some time to actually explore the planet.

A Hohmann Transfer Orbit is the most fuel-efficient (but not time-efficient) method of transferring between two coplanar orbits. Mars and Earth orbit on the same plane (as in fact do all the planets in the Solar System), and therefore a Hohmann Transfer Orbit is applicable in this scenario. Essentially, given that the two planets have different periods of orbit, we need to find the initial position of Mars so that when we arrive, it is at point A below:



A is the aphelion (point furthest from the Sun) and P is the perihelion (closest to the Sun) of the Hohmann Transfer Orbit. Note that the length of AP is ($r_1 + r_2$).

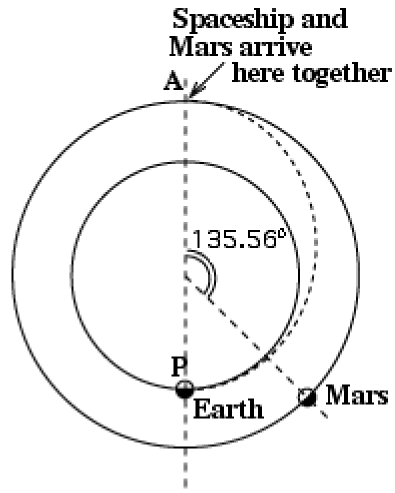
Note that the Sun is not depicted, and also note that the orbits of Earth (inner circle) and Mars (outer circle) are roughly circular.

From Kepler's Third Law we know that $\frac{T^2}{a^3} = \boldsymbol{constant,}$ where **T** is period of orbit and **a** is the semi-major axis length (half of length AP in this case). By using units of **T** as years and units of **a** as AU (astronomical units) then we can work out this constant for Earth (and then apply this to all other bodies orbiting the Sun, including Mars and our Transfer Orbit). The length of one orbit of the Earth around the Sun is 1 year, and the distance in AU is 1 AU (by definition). Therefore the constant is $\frac{1}{1} = 1$.

Applying Kepler's Law to the Hohmann Transfer Orbit:

$$\frac{T^2}{\boldsymbol{a}^3} = \mathbf{1}$$
$$T^2 = \boldsymbol{a}^3$$

a = semi-major axis length, which in this case is AP/2. Length AP = r_1+r_2, which in AU is $1 + 1.523691 = 2.5236911$AU.
Dividing this by two gives **a** = 1.261845.
So $\mathbf{a}^3$ = 2.00918 = T^2



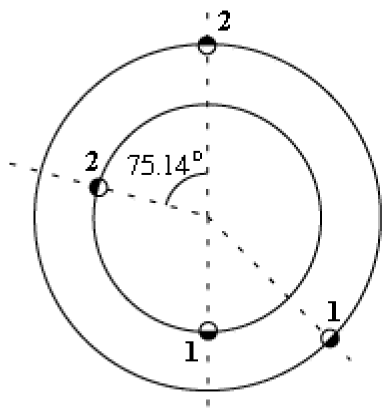
Square rooting gives T, which is 1.417454 (years). Note that this is the time period for a COMPLETE orbit – we are only going half-way. This therefore becomes 0.70873 years or about 259 days. This is the travel time to Mars.

Note that the orbital period of Mars is 1.8822 Earth years, so during the time of travel from Earth to Mars, the change in angle of Mars from the Sun is

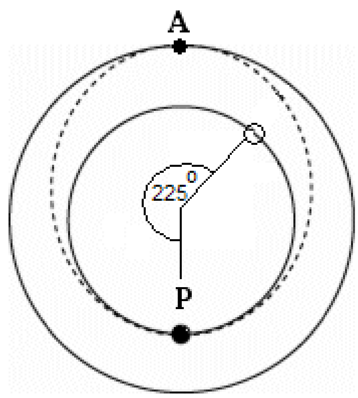
$$\mathbf{360^o \text{ x } (0.70873/1.8822) = 135.6^o}$$

We must therefore launch when Mars is 135.6^o from point A, or 44.4^o 'in front' of Earth's orbit.

Keeping in mind that it takes 0.70873 years to arrive, the new position of the Earth and Mars is shown on the right, where 1 is position at time of launch and 2 is position at time of arrival. Note that Mars is no longer 'ahead' of the Earth and Earth is now 'ahead' of Mars in orbit.



Taking the exact route home means that the position of the planets must look like on the right (we are travelling from A to P), where P is the final position of the Earth when arriving from Mars, and A is the position of Mars when leaving from Mars. This will take 0.70873 years, as before, and in this time the Earth will have swept out 360^o x (0.70873/1) degrees, i.e. 255.1^o. The Earth must therefore be at the hollow circle when launched, i.e. it is behind **A** (Mars) by 75.1^o. Comparing this with the previous diagram shows that we cannot instantly take off again – there must be some period of waiting for the "ideal" moment.



Let T be the amount (in years) that the astronauts stay. In this time x, Earth will have swept out 360^o x (T/1) and Mars will have swept out 360^o x (T/1.8822). If T is 0, the astronauts are leaving immediately after arrival and the positioning of the planets is incorrect (Mars should be ahead by 75.1^o; instead, Earth is ahead by 75^o. These angles are very similar only by coincidence).

Angle of Mars (relative to 12-o'clock) is 360^o x (T/1.8822).
Angle of Earth (relative to 12-o'clock) is 360^o x (T/1) + 75^o.

Angle of Mars – Angle of Earth = 75.1 for optimal time of depature, so:
360T/1.8822 - 360T + 75 = 75.1 + 360n (the extra whole number of revolutions is a result of the Earth orbiting faster than Mars. N=0 gives a negative answer so n=1 has been used here.)
360T – 1.8822 x 360T = 360.1 x 1.8822
T x (360 - 1.8822 x 360) = 360.1 x 1.8822
T = 2.13415 years.

However, we do not believe that astronauts can stay for this long (and doing so would require far too much food/supplies). Instead, we have chosen for the astronauts to stay for 38.5 days so that they can explore Mars and conduct experiments to a good extent. This value is obviously easily changed; we have chosen this for convenient calculations in further steps.

The trip back to Earth will take slightly less time as the astronauts are no longer in the optimal position (the calculation works out to be 42/365 / 1.8822 x 360 – 360 x 42/365 + 75.1 = 55.7^o, which is not ahead of Earth by enough. The Earth will therefore surpass the spacecraft if the speed is not adjusted – the spacecraft therefore needs to travel faster than the return journey back, using more fuel, but saving time. Because the Earth will also have moved ahead slightly (in overtaking the space shuttle), the total distance will also increase. Depending on the chosen increase in speed, the increase in distance also varies. We have chosen for the increase in speed and increase in distance to be roughly by the same scale factor – this means the return journey takes the exact same amount of time. In any case, this has little bearing on the (more important) travel to Mars, with the changes being around 1% or so.

2. What will the astronaut need, and more importantly, what is the mass of these resources?

Firstly we must consider what a mission to Mars will require:

- A means of getting there
- Food, Water, Air as necessities
- Scientific Equipment

Firstly, we have decided to have one astronaut on this mission (who is trained for the psychological effects of operating by him/herself for the 553 day mission. We have decided to take the average weight for a person to be based on the mean weight of a male American aged 20 or over, as this is the most likely bracket from which the astronaut will be found. According to a report based on US National Health Surveys, we take the average weight to be **86.1kg**[1].

[1] *Mean Body Weight, Height, and Body Mass Index United states 1960-2002* from http://www.cdc.gov/nchs/data/ad/ad347.pdf

Using statistics from NASA[2], we took the following constants:

Amount of food, water, air required each day = **30.60kg per person**
Amount of food required each day = **1.83kg per person**
Amount of air and water required each day = 30.60 – 1.83 = **28.77kg per person**

The ISS has a recycling system for water that is 98% efficient[3]. We think a similar system should be used for our Mars mission.

With efficiency comparable to the ISS, the amount of water can be modelled as,

$$mass\ of\ water, W = \frac{28.77}{0.98^n}$$, for n number of days.

If the mission lasts n = 500 days, then the water needed to be carried is:

$$W = \frac{28.77}{0.98^{500}}$$ =701297kg, which is a significant amount.

To make a mission to Mars realistic, we need to assume that there will be a system with efficiency of recycling to be very close to 100%. That way we only need 100kg of water, a round figure that will account for minor accidental losses.

Also, with waste produced by the astronaut, we assume that the amount of food consumed equals the amount of waste produced (that is unrecyclable). With this in mind, we can hence model the mass required for essentials with:

$$Mass\ of\ essentials\ on\ the\ space\ craft\ after\ x\ days\ with\ total\ duration\ of\ mission\ n$$
$$= 1.83(n - x) + 100 - 1.83x$$

At launch, mass of essentials is **1112kg**.

Upon arrival to Mars we have decided that the astronaut will require the use of a vehicle similar to the capabilities of the lunar rover. Considering the Lunar Electric Rover (LER) as a vehicle that should be of similar configuration to any Mars-bound vehicle, we have added **4000kg** to the Mars mission based on the specifications available[4]. This should cover any equipment an astronaut would require.

The actual spacecraft we have chosen for the Mars mission is a craft based on the Shenzhou. We have not gone into specifics for the actual technical capabilities of the vessel, but are instead more interested in its mass. We take the mass of our craft to be **7840kg**[5].

Using the Apollo 11 mission as a reference we have chosen the specifications (i.e. mass) of the Saturn V rocket as our template for the rocket that will allow the astronaut to escape Earth. We take its dry mass, that is, without fuel, to be **118000kg**[6].

[2] http://www.nasa.gov/audience/foreducators/stseducation/materials/Sustaining_Life.html
[3] http://www.vscht.cz/document.php?docId=2805
[4] http://www.nasa.gov/pdf/284669main_LER_FactSheet_web.pdf
[5] http://www.braeunig.us/space/specs/shenzhou.htm
[6] http://en.wikipedia.org/wiki/Mass_ratio

Overall, the mass of our rocket and its payload before take-off, and without fuel:

$$mass = 86.1 + 4000 + 7840 + 118000 + 1112 = \mathbf{131038.1kg}$$

3. How much fuel will we need to achieve the mission?

As we consider the main question, we first need to consider different methods of propulsion. In investigating the idea of specific impulse as a measure of a propulsion system's effectiveness we found the following data:

Engine	Effective exhaust velocity (m/s, kg·m/s/kg)	Specific impulse (s)	Energy per kg of exhaust (MJ/kg)
Turbofan jet engine	*29,000*	*3,000*	*~0.05*
Solid rocket	*2,500*	*250*	*3*
Bipropellant liquid rocket	*4,400*	*450*	*9.7*
Ion thruster	29,000	3,000	430
Dual Stage Four Grid Electrostatic Ion Thruster	210,000	21,400	22,500
VASIMR	290,000	30,000	43,000

Table 1: Specific Impulse and other related data for modes of propulsion[7]

Of the engine types we decide to adopt only systems that were flight proven[8] in order for our calculations to be reasonably plausible. From the above table only the Turbofan, Solid and Bipropellant systems were proven and as such we selected a Bipropellant liquid rocket as the basis for our calculations on grounds that it released the greatest amount of energy per kg of exhaust.

In looking at the types of fuel we considered Liquid hydrogen and RP-1 (based on kerosene) which are combined with Liquid Oxygen for propulsion. Liquid hydrogen, whilst being efficient in its combustion, requires cold temperatures for storage and a large volume due to its low density[9]. On the other hand, RP-1 is cheaper, denser and far less dangerous[10]. Because our chosen model, the Saturn V uses of RP-1, we chose RP-1 as our fuel.

[7] From http://en.wikipedia.org/wiki/Specific_impulse (adapted)
[8] http://en.wikipedia.org/wiki/Spacecraft_propulsion#Table_of_methods
[9] http://en.wikipedia.org/wiki/Liquid_hydrogen
[10] http://en.wikipedia.org/wiki/RP-1, http://www-pao.ksc.nasa.gov/kscpao/nasafact/count2.htm

Pre Calculations:

For the purposes of our investigation we have split the mission into 4 stages as shown below:

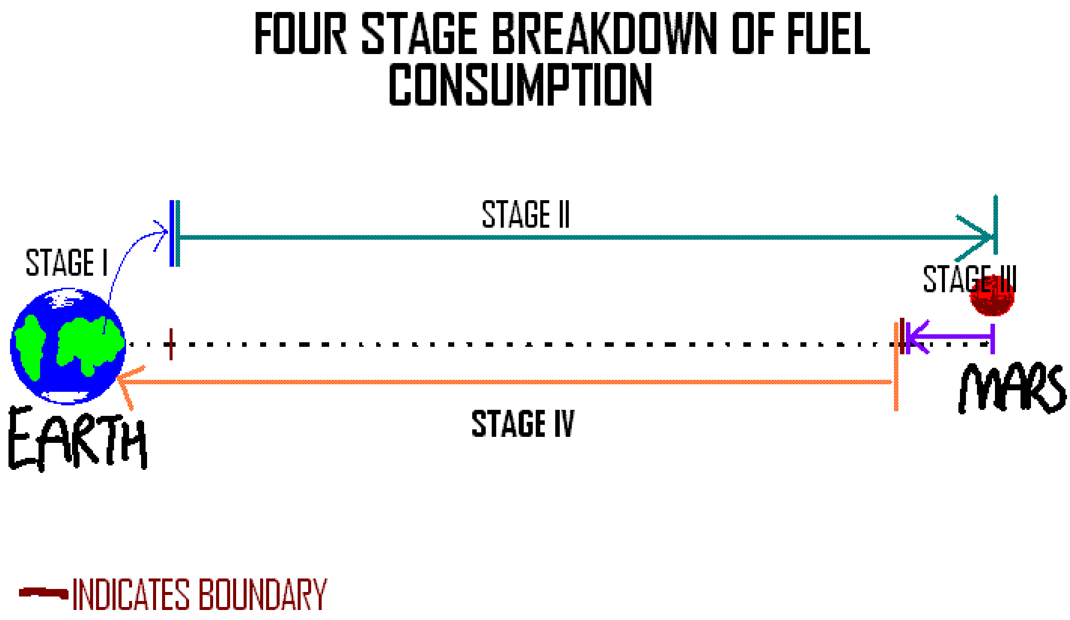


Figure 1: Breakdown of fuel consumption

Stage I: Getting from the Surface of Earth to the Boundary between the Earth's atmosphere and Outer Space

The potential at the surface of the Earth is calculated thus:

$R = radius\ of\ Earth = 6.371 \times 10^6$
$M = mass\ of\ Earth = 5.98 \times 10^{24}$

We assume that the Earth is a point mass and that the surfacee is a point R metres from the point mass.

Then, the potential = $\varphi = -\frac{GM}{R} = \frac{-6.67\times10^{-11}\times5.98\times10^{24}}{6.371\times10^6} = 6.260610 \times 10^7\ \text{Jkg}^{-1}$

Hence the Energy per unit mass required to get the rocket up there is:

$$\text{Energy per unit mass} = 6.260610 \times\ 10^7$$

Using the Saturn V rocket Stage I as our model, the fuel burnt is Liquid Oxygen with Kerosene (RP-1).

$$Energy\ per\ kg\ of\ exhaust = energy\ per\ kg\ of\ fuel\ mix = 9.7\text{MJkg}^{-1}$$

(using the Law of Conservation of Mass and Energy)

Thus, 1 kg of LOx/RP-1 mixture can be used to propel:

$$\frac{9.7\times10^{7}}{6.260610\times10^{7}} = 1.54936979 \text{ kg} \text{ of mass to escape earth's field.}$$

This will be our standard value for further calculations.

For this calculation, we have assumed that air resistance is negligible, that the fuel will have a constant value of 9.7MJkg^{-1} and that the conversion of chemical energy into kinetic energy is 100% efficient.

This means that the final value that we get for the fuel used may be lower. In real applications, we must consider each of the above assumptions.

Stage II: Getting from the Boundary of Earth's Atmosphere to Mars

Referring to the calculations done previously, the amount of fuel needed for the boost it out of the Earth's orbit and into the Sun's orbit to Mars is 12.37kg. This will be the same value we use for Stage IV, which is the reverse, getting from Mars back to Earth.

$\omega_{\text{earth}} = 2.0 \times 10^{-7}$ $\quad r_{earth\ to\ sun} = 1.5 \times 10^{11}$ $\quad v = 30000ms^{-1}$

$\omega_{\text{transfer orbit}} = 2.8 \times 10^{-7}$ $\quad r_{transfer\ orbit} = 1.89 \times 10^{11}$ $\quad v = 52900\ ms^{-1}$

$\omega_{\text{mars}} = 1.05 \times 10^{-7}$ $\quad r_{mars\ to\ sun} = 2.28 \times 10^{11}$ $\quad v = 24000ms^{-1}$

For the spacecraft to leave the orbit around the Earth to enter the transfer orbit around the Sun, it must exceed 30000ms^{-1} which is the velocity of the Earth around the Sun (otherwise it will become a satellite around the Earth). We assume that the transfer orbit is circular with a radius half way between that of Mars and the Earth (because Mars is 1.5 times the distance of the Earth from the Sun, the eccentricity of the orbit is low and therefore it can be approximated to be circular).

The velocity needed to enter into the transfer orbit is 52900ms^{-1}, from above. Therefore it must accelerate to 52900ms^{-1} from 29800ms^{-1}, which we assume happens almost instantaneously (as a burst of energy) as it enters into the transfer orbit. From then on, the gravitational force from the sun will cause the spacecraft to carry on in its natural path towards Mars. It will decelerate to the same tangential speed of Mars (due to Kepler's Second Law).

As it approaches Mars, it needs to decelerate so it will become taken by Mars' orbit (i.e. it must leave its orbit around the Sun to become a satellite around Mars). If it does not leave the Sun's orbit, it will overshoot the planet and curve back (in its orbit around the Sun). It will be at 24000ms^{-1} as a result of Kepler's Law. However, it must then decrease down to 0ms^{-1} so that the shuttle can land. This will take some time and will take two hours (like the Apollo 11) which will require an average deceleration of 3.33ms^{-1}. The rocket would be roughly 100000kg (in fact less, as some fuel has been lost on the journey). This would require around 100m and need approximately 3.4kg of fuel.

Stage III: To escape the gravitational field of Mars

Now, our astronaut has landed on Mars and has done the experiments needed, in the time frame of 38.5 days, as calculated.

Now we need to find the mass of fuel required to escape from the Martian gravitational field.

Mass of Mars = M = 6.4185*10^23 kg
Radius of Mars = R = 3389.5 km = 3.3895*10^6 m

So, the Potential at the surface of Mars is given by

$$\varphi = -\frac{GM}{R} = -1 * 6.67 * 10^{\wedge} - 11 * \frac{6.4185 * 10^{\wedge}23}{3.3895 * 10^{\wedge}6} = -\ 12630593 \text{ J kg}^{-1}$$

Hence, the Energy per unit mass required to make the rocket escape from the gravitational field of mars is:

$$Energy\ per\ unit\ mass = 12630593 \text{ J kg}^{-1}$$

Using the same fuel (Lox/RP-1), the ratio is now:

9.7*10^7/1.2630593*10^7 = 7.679766104

Which is approximately = 7.7 kg can be lifted by 1 kg of fuel on Mars.

Similarly, we have assumed that air resistance is negligible, that the fuel will have a constant value of 9.7 MJ $\text{kg}^{-1}$and that the conversion of chemical energy into kinetic energy is 100% efficient.

Calculating the amount of fuel that we need:

Firstly, we define the function T(x). This models the mass of fuel left after x days.

Engine	Effective exhaust velocity (m/s, kg·m/s/kg)	Specific impulse (s)	Energy per kg of exhaust (MJ/kg)
Turbofan jet engine	*29,000*	*3,000*	*~0.05*
Solid rocket	*2,500*	*250*	*3*
Bipropellant liquid rocket	*4,400*	*450*	*9.7*
Ion thruster	29,000	3,000	430
Dual Stage Four Grid Electrostatic Ion Thruster	210,000	21,400	22,500
VASIMR	290,000	30,000	43,000

Table 2: Specific Impulse and other related data for modes of propulsion[11]

Table of the mass of fuel remaining in the rocket:

At day 0, there is the maximum amount of fuel (initial amount)

[11]From http://en.wikipedia.org/wiki/Specific_impulse (adapted)

We cannot calculate this yet.

It takes about 8 minutes for the space shuttle to reach the outer atmosphere[12]. This is represented as T=1.

Hence,

$$T(1) = T(0) - \frac{131038 - 1.83 + T(0)}{1.54936979}$$

Our suggestion for Stage III is that there will be a rapid acceleration (because it occurs over 1mm) once we have escaped the earth's atmosphere to get to Mars, and allow the natural forces (via Kepler's Second Law) to decelerate us enough to get to Mars. The amount of energy required for this stage is **120MJ.**

Hence, the mass of fuel used is: $\frac{120}{9.7}$ = **12.37kg**

So, T(296) = T(1) - (12.37)

Between Stages II and III is the **38.5** day stay on Mars

So, T(335)= T(296)

Stage III is where the rocket must escape Mars' gravitational field.
The loss in mass:

$$T(335) = T(296) - \frac{131038 - 1.38 \times 335 + T(296)}{1.54936979}$$

Stage IV is a similar rapid acceleration. So, subtract **12.37kg**
When we land on earth, the fuel left is 0.
Given that T(553) = T(335) – 12.37 = 0
We can now work backwards to find T(0)

$$T(335) = 12.37 = T(296) - \frac{131038 - 1.38 \times 355 + T(296)}{7.7}$$

Therefore, T(296) =2545.2436

So, $2545.2436 = T(1) - 12.37$

Therefore $T(1) = 2545.2436 + 12.37 = 2557.6136$

$$T(1) = 2557.6136 = \mathrm{T}(0) - \frac{131038 - 1.83 + \mathrm{T}(0)}{1.54936979}$$

Therefore **T(0) = 158602.5776 kg**

[12] http://wiki.answers.com/Q/How_long_will_it_take_a_rocket_to_get_into_outer_space_from_the_earth_surface

Conclusion

In conclusion, it is indeed possible to travel to Mars. Using a similar fuel to the Apollo 11 and Saturn V (but on a larger scale) we were able to calculate that 158600kg of fuel would be needed to travel to Mars and back. Using an elliptical transfer orbit the space shuttle would minimise the fuel needed to travel to Mars – this journey would take 258 days to get there, allowing for 38 days to investigate the planet, and 258 days to travel back. This gives a total of 553 days of travel, in line with many current predictions.

In our models we had to make many simplifications, like approximating elliptical orbits with very low eccentricity (like Mars and Earth, and to a greater extent the transfer orbital route) as circles with constant radii. We assumed conservation of mass, 100% efficiency of fuel as well as recycling water, and that there was no air resistance going into the atmosphere. The time given to create these models was insufficient to allow for detailed analysis of elliptical orbits and compounding rates of fuel loss, as well as inefficient water recycling and fuel combustion. With more time, we would also have been able to do a computer simulation of the journey modelled with rates of fuel loss. We would also have liked to consider other orbit pathways like those taking less time (but more fuel).

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